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Random fixed point theorem in polish spaces

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ABSTRACT : In the present paper we establish a fixed point theorem for random operator in polish spaces at the basis of complete metric spaces.

Keywords : Fixed point, Polish space, random operator

AMS Subject Classification: 47H10,H25

I. INTRODUCTION AND PRELIMINARIES

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. In recent years, the study of random fixed point have attracted much attention, some of the recent literatures in random fixed points may be noted in [3,5,6,7,8,9]. In particular random iteration schemes leading to random fixed point of random operator.

The present paper deals with some fixed point theorems in Polish spaces for random operator at the basis of complete metric spaces.

Throughout this paper, (Ω, Σ) denotes a measurable space, X be a complete metric space and C is non empty subset of X.

Definition 1: A function $f : \Omega \to C$ is said to be measurable if $f'(B \cap C) \in \Sigma$ for every Borel subset B of X.

Definition 2: A function $f : \Omega \times C \to C$ is said to be random operator, if $f(., X) : \Omega \to C$ is measurable for every $X \in C$.

Definition 3: A random operator $f : \Omega \times C \to C$ is said to be continuous if for fixed $t \in \Omega$, $f(t, .) : C \times C$ is continuous.

Definition 4: A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator $f : \Omega \times C \to C$, if $f(t, g(t)) = g(t)$, $\forall\, t \in \Omega$.

III. MAIN RESULTS

Theorem 1: Let S and T from $\Omega \times X \to X$ be two continuous random multivalued operator of a Polish spaces (X, d), such that

$$d(S(w,x),T(w,y)) < d(x,y), x \neq y \in X.$$

and there exists a subset $A \subset X$ and a point $x_0 \in A$ satisfying the following conditions:

(*i*) $d(x_0, S(w, x)) - d(S(w, x_0), ST(w, x)) \geq 2d(x_0, S(w, x_0))$ and $ST = TS$

(*ii*) $d(S(w, x), T(w, y)) \leq \alpha\, d(x, y)$

$$\{d(x, S(w,x)), d(y, T(w,y))\}^{1/2}$$
$$+\beta\, d(x,y)\{d(x, T(w,y)), (y, S(w,x))\}^{1/2},$$

for $x, y \in A$,

Where α and β be is a measurable mapping $\alpha, \beta : \Omega \to (0, 1)$ and $w \in \Omega$ be a selector.

Then there exists a unique common fixed point of S and T.

Proof : Let $\xi_0 : \Omega \times X$ be an arbitrary measurable mapping and assume that

$\xi_0(w) \neq S(w, \xi_0(w))$ and let a sequence $\{\xi_n(w)\}$ such that

$S(w, \xi_0(w)) = \xi_1(w)$, $T(w, \xi_1(w)) = \xi_2(w)$ ...., $S(w, \xi_{2n}(w)) = \xi_{2n+1}(w)$,

$T(w, \xi_{2n+1}(w)) = \xi_{2n+2}(w)$.

From $d(S(w, \xi_1(w)), T(w, \xi_2(w)) < d(\xi_1(w), \xi_2(w))$

The Sequence $d(\xi_n(w), \xi_{n+1}(w))$ is non increasing and

$d(\xi_n(w), \xi_{n+1}(w)) < d(\xi_0(w), \xi_1(w))$, for $n = 1, 2, 3, ...$

Now from the triangle inequality and $ST = TS$,

$$d(\xi_0(w), \xi_{2n+1}(w)) \leq d(\xi_0(w), \xi_1(w)) + d(\xi_1(w), \xi_{2n+2}(w)) + d(\xi_{2n+2}(w), \xi_{2n+1}(w))$$

$$= d(\xi_0(w), \xi_1(w)) + d(S(w), \xi_0(w)), TS(w, \xi_{2n}(w)) + d(\xi_{2n+2}(w), \xi_{2n+1}(w))$$

Thus

$$= 2d(\xi_0(w), \xi_1(w)) + d(S(w), \xi_0(w)), TS(w, \xi_{2n}(w))$$

$$d(\xi_0(w), S(w, \xi_{2n}(w)) - d(S(w, \xi_0(w)), TS(w, \xi_{2n}(w)) < 2d(\xi_0(w, S(w, \xi_0(w))$$

Hence from the condition (*i*), if follows that $\xi_{2n+1}(w) \in A$ for each n.

Similarly $\xi_{2n+1}(w) \in A$ for each n.

Therefore $\xi_{2n+2}(w) \in A$ for each n.

Now, we show that the sequence $\{\xi_{2n}(w)\}$ is bounded. For this consider

$$d(\xi_0(w), \xi_{2n+2}(w)) \leq d(\xi_0(w), S(w, \xi_0(w)) + d(S(w, \xi_0(w)), T(w, \xi_{2n+1}(w)) + d(S(w, \xi_{2n}(w)), T(w, \xi_{2n-1}(w))$$

$+d(\xi_{2n+1}(w), \xi_{2n+2}(w))$
$\leq 3d(\xi_0(w), S(w), \xi_0(w)) + \alpha d(\xi_0(w), \xi_{2n-1}(w))$
$\{d(\xi_0(w), \xi_1(w)d, \xi_{2n+1}(w)), \xi_{2n}(w))\}^{1/2}$
$+ \beta d(\xi_0(w), \xi_{2n+1}(w)) \{d(\xi_0(w), \xi_{2n}(w))\, d(\xi_{2n-1}(w), \xi_1(w))\}^{1/2}$

$\leq \{3 + \alpha d(\xi_0(w), \xi_{2n-1}(w))$
$+ \beta d(\xi_0(w), \xi_{2n-1}(w))\}\, d(\xi_0(w), \xi_1(w))$.

Hence for a given $d_1 > 0$ with $d(\xi_0(w), \xi_{2n-1}(w)) \geq d_1$, we get

$d(\xi_0(w), \xi_{2n+2}(w)) \leq \{3 + \alpha d_1 + \beta d_1\}\, d(\xi_0(w), \xi_1(w))$.

Similarly we can show that

$d(\xi_0(w), \xi_{2n+1}(w)) \leq \{2 + \alpha d_1^* + \beta d_1^*\}\, d(\xi_0(w), \xi_1(w))$,

Where for a given $d_1^* > 0$,

$d(\xi_0(w), \xi_{2n+1}(w)) \leq d_1^*$

Hence $\{\xi_n(w)\}$ is bounded.

By continuous calculation, it follows that for each n, $d(\xi_n(w), \xi_{n+1}(w)) \leq \{(\alpha^2 + \beta^2)\, d(\xi_{n-1}(w), \xi_n(w))\, (\alpha^2 + \beta^2)$ $d(\xi_{n-2}(w), \xi_{n-1}(w)) \ldots (\alpha^2 + \beta^2)\, d(\xi_0(w), \xi_1(w))\}$ $d(\xi_0(w), \xi_1(w))$

Let $\in > o$. If $d(\xi_i(w), \xi_{i+1}(w)) \geq \in$, for $i = 0, 1, 2, \ldots.$, then

$(\alpha^2 + \beta^2)\, d(\xi_i(w), \xi_{n+1}(w)) \leq (\alpha^2 + \beta^2)(\in)$, for $i = 0, 1, 2 \ldots.$ and also $o \leq (\alpha^2 + \beta^2)(\in) < 1$.

Therefore,

$d(\xi_n(w), \xi_{n+1}(w)) \leq \{(\alpha^2 + \beta^2)(\in)\}^n\, d(\xi_0(w), \xi_1(w))$.

Which proves that $\{\xi_n(w)\}$ is a Cauchy sequence.

As X is complete, $\lim_{n\to\infty} \xi_n(w) = \eta \in X$.

Using continuity of S and T, we find that η is a common fixed point of S and T. This ends the proof.

Corollary : If we put $A = X$ and if.

$d(S(w), \xi_2(w), T((w, \xi_2(w)) \leq \alpha\, d(\xi_1(w), \xi_2(w))$
$\{d(\xi_1(w), S(w, \xi_1(w))\, d(\xi_2(w), T(w, \xi_2(w))\}^{1/2}$
$+ \beta d(\xi_1(w), \xi_2(w))\, \{d(\xi_1(w), T(w, \xi_2(w))$
$d(\xi_2(w), S(w, \xi_1(w))\}^{1/2}$

$\forall\, \xi_1(w), \xi_2(w) \in X$, then S and T have a unique common fixed point.

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